2.4 STANDARD CHARGE CONFIGURATIONS

In three special cases the integration discussed in Section 2.3 is either unnecessary or easily carried out. In regard to these standard configurations (and to others which will be covered in this chapter) it should be noted that the charge is not "on a conductor." When a problem states that charge is distributed in the form of a disk, for example, it does not mean a disk-shaped conductor with charge on the surface. (In Chapter 6, conductors with surface charge will be examined.) Although it may now require a stretch of the imagination, these charges should be thought of as somehow suspended in space, fixed in the specified configuration.

Point Charge

As previously determined, the field of a single point charge Q is given by

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\mathbf{a}_r \qquad \text{(spherical coordinates)}$$

See Fig. 2-2(*a*). This is a spherically symmetric field that follows an *inverse-square law* (like gravitation).

Infinite Line Charge

If charge is distributed with *uniform* density ρ_ℓ (C/m) along an *infinite, straight* line—which will be chosen as the z axis—then the field is given by

$$\mathbf{E} = \frac{\rho_\ell}{2\pi\epsilon_0 r}\mathbf{a}_r \qquad \text{(cylindrical coordinates)}$$

See Fig. 2-6. This field has cylindrical symmetry and is inversely proportional to the *first power* of the distance from the line charge. For a derivation of **E**, see Problem 2.9.

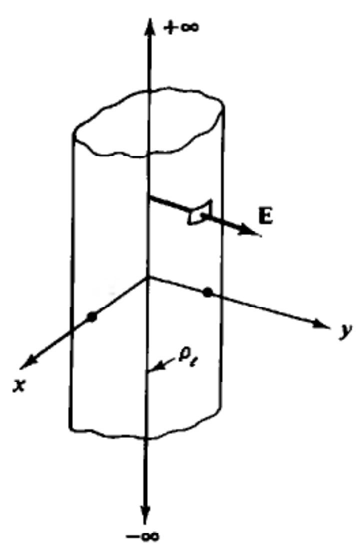

Fig. 2-6

EXAMPLE 3. A uniform line charge, infinite in extent, with $\rho_\ell = 20$ nC/m, lies along the z axis. Find **E** at (6, 8, 3) m.

In cylindrical coordinates $r = \sqrt{6^2 + 8^2} = 10$ m. The field is constant with z. Thus

$$\mathbf{E} = \frac{20\times10^{-9}}{2\pi(10^{-9}/36\pi)(10)}\mathbf{a}_r = 36\mathbf{a}_r \text{ V/m}$$

Infinite Plane Charge

If charge is distributed with *uniform* density ρ_s (C/m²) over an *infinite plane*, then the field is given by

$$\mathbf{E} = \frac{\rho_s}{2\epsilon_0}\mathbf{a}_n$$

See Fig. 2-7. This field is of constant magnitude and has mirror symmetry about the plane charge. For a derivation of this expression, see Problem 2.12.

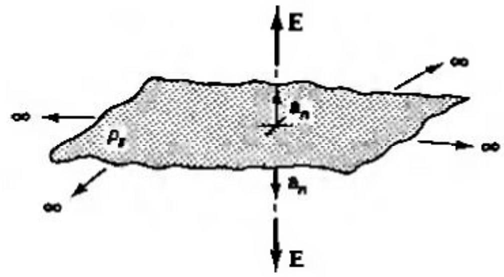

Fig. 2-7

EXAMPLE 4. Charge is distributed uniformly over the plane $z = 10\,\text{cm}$ with a density $\rho_s = (1/3\pi)\,\text{nC/m}^2$. Find $\mathbf{E}$.

$$|\mathbf{E}| = \frac{\rho_s}{2\epsilon_0} = \frac{(1/3\pi)10^{-9}}{2(10^{-9}/36\pi)} = 6\ \text{V/m}$$

Above the sheet ($z > 10\,\text{cm}$), $\mathbf{E} = 6\mathbf{a}_z$ V/m; and for $z < 10\,\text{cm}$, $\mathbf{E} = -6\mathbf{a}_z$ V/m.

Solved Problems

2.1. Two point charges, $Q_1 = 50\,\mu\text{C}$ and $Q_2 = 10\,\mu\text{C}$, are located at $(-1, 1, -3)$ m and $(3, 1, 0)$ m, respectively (Fig. 2-8). Find the force on Q_1.

$$\mathbf{R}_{21} = -4\mathbf{a}_x - 3\mathbf{a}_z$$

$$\mathbf{a}_{21} = \frac{-4\mathbf{a}_x - 3\mathbf{a}_z}{5}$$

$$\mathbf{F}_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2}\mathbf{a}_{21}$$

$$= \frac{(50\times10^{-6})(10^{-5})}{4\pi(10^{-9}/36\pi)(5)^2}\left(\frac{-4\mathbf{a}_x - 3\mathbf{a}_z}{5}\right)$$

$$= (0.18)(-0.8\mathbf{a}_x - 0.6\mathbf{a}_z)\ \text{N}$$

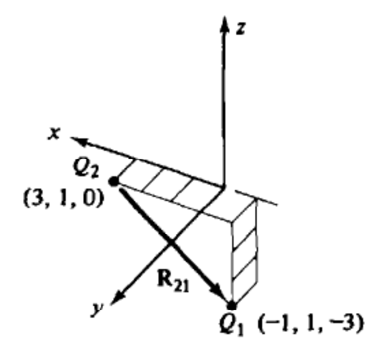

Fig. 2-8